Exercise 93

Use the Chain Rule to prove the following.

(a) The derivative of an even function is an odd function.

(b) The derivative of an odd function is an even function.

Solution

Part (a)

Suppose that $f(x)$ is an even function: $f(-x) = f(x)$. Consider the derivative of $f(x)$ and then use the chain rule.

$$\begin{aligned} f'(x) &= \frac{df}{dx} \\ &= \frac{d}{dx}[f(x)] \\ &= \frac{d}{dx}[f(-x)] \\ &= f'(-x) \cdot \frac{d}{dx}(-x) \\ &= f'(-x) \cdot (-1) \\ &= -f'(-x) \end{aligned}$$

Since $f'(x) = -f'(-x)$, the derivative of $f(x)$ is an odd function.

Part (b)

Suppose that $f(x)$ is an odd function: $f(-x) = -f(x)$. Consider the derivative of $f(x)$ and then use the chain rule.

$$\begin{aligned} f'(x) &= \frac{df}{dx} \\ &= \frac{d}{dx}[f(x)] \\ &= \frac{d}{dx}[-f(-x)] \\ &= -f'(-x) \cdot \frac{d}{dx}(-x) \\ &= -f'(-x) \cdot (-1) \\ &= f'(-x) \end{aligned}$$

Since $f'(x) = f'(-x)$, the derivative of $f(x)$ is an even function.